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MATHEMATICAL MODELLING USING METHODS OF GETTING PARTIAL DIFFERENTIAL EQUATION MODELS

Dr. Swati Agarwal

Assistant Professor

Department of Mathematics

Mangalayatan University, Aligarh, (Uttar Pradesh)

Abstract

Mathematical Model is a representation in mathematical terms of the behavior of real gadgets and objects. The modeling of gadgets and phenomena is essential to both engineering and science; specialists and researchers have practical reasons for doing mathematical modeling. Partial differential equation (PDE) models arise when the variables of intrigue are functions of more than one independent variable and all the dependent and independent variables are nonstop. In this paper we discuss about mass of a volume component increased by its speeding up Vector is equivalent to the vector aggregate of all the outside body forces following up on the volume component and the internal forces because of the activity of the remainder of the substance on the volume component viable, we get straightforwardly a partial differential condition.

Keywords: partial differential equation, mathematical, modeling, models, etc.

1. INTRODUCTION

Mathematical Model is a representation in mathematical terms of the behavior of real gadgets and objects we want to realize how to make or generate mathematical representations or models, how to validate them, how to utilize them, and how and when their utilization in restricted. Be that as it may, before diving into these important issues, it merits talking about why we do mathematical modeling. Since the modeling of gadgets and phenomena is essential to both engineering and science, specialists and researchers have practical reasons for doing mathematical modeling. In addition, designers, researchers, and mathematicians want to encounter the sheer delight of formulating and tackling mathematical issues. The momentum-balance rule as Newton's second law viz. On the off chance that we apply the mass of a volume component increased by its acceleration Vector is equal to the vector total of all the external body forces acting on the volume component and the internal forces because of the action of the remainder of the substance on the volume component viable, we get legitimately a partial differential equation.

A few Characteristics of Mathematical Models:

i. **Realism of Models:** We want a mathematical model to be as realistic as conceivable and to speak to reality as intently as conceivable. Be that as it may, if a model is extremely realistic, it may not be mathematically tractable. In making a mathematical model, there has to be a tradeoff among tractability and reality.

ii. **Hierarchy of Models:** Mathematical modeling is certainly not a one-shot affair. Models are constantly improved to make them progressively realistic. Along these lines for each situation, we get a hierarchy of models, each more realistic than the former and each liable to be trailed by a superior one.

iii. **Relative Precision of Models:** Different models contrast in their precision and their agreement with observations.

iv. **Robustness of Models:** A mathematical model is said to be vigorous if small changes in the parameters lead to small changes in the behavior of the model. The choice is made by utilizing affectability analysis for the models.

v. **Self-consistency of Models:** A mathematical model included equations and in equations and the must be steady, for example a model cannot have both $x+y>a$ &$x+y<a$. Some of the time the irregularity results from irregularity of basic assumptions. Since mathematical irregularity is relatively easier to discover, this gives a technique for discovering irregularity in prerequisites which social or biological researchers may expect of their models. A well known example of this is given by Arrow's Impossibility Theorem.

Partial differential equation (PDE) models arise when the variables of intrigue are functions of more than one independent variable and all the dependent and independent variables are nonstop. Along these lines in fluid dynamics, the velocity segments u, v, w and the weight p at any point x, y, z and at any time t are functions of x, y, z, t and in general u(x, y, z, t), v(x, y, z, t), w(x, y, z, t), p(x_t y, z, t) are consistent functions, with persistent first and second order partial derivatives, of the constant independent variables x, y, z, t. Similarly the electric field intensity vector $\vec{E}$ (x, y, z, t), the magnetic field intensity vector $\overrightarrow{H}$(x, y, z, t), the electric current density vector $\vec{J}$ (x, y, z, t), the temperature T(x, y, z, t) and the displacement vector $\vec{D}$(x, y, z, t); of an elastic substance are in general ceaseless vector or scalar functions with constant derivatives.

2. NAVIER-STOKES EQUATIONS FOR THE FLOW OF A VISCOUS INCOMPRESSIBLE FLUID

Let u (x, y, z, t), v (x, y, z, t), w (x, y, z, t) and p (x, y, z, t) mean individually the three velocity segments and weight at the point (x, y, z) at time t in a fluid with constant density p and thickness coefficient µ. Then the equation of progression, which communicates the fact that the amount of fluid entering a unit volume for each unit time is the same as the amount of the fluid leaving it per unit time, is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

The equations of motion, known as Navier-Stakes equations, for the flow of a Newtonian viscous incompressible fluid are

$$p\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=X-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (2)$$

$$p\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=Y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \quad (3)$$

$$p\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=Y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \quad (4)$$

In the event that the outside body forces X, Y, Z structure a traditionalist framework, there exists a potential function Ω to such an extent that

$$X=-\frac{\partial\Omega}{\partial x},\ Y=-\frac{\partial\Omega}{\partial y},\ \ Z=-\frac{\partial\Omega}{\partial z}, \quad (5)$$

$$X-\frac{\partial p}{\partial x}=-\frac{\partial}{\partial x}(\Omega+p),\ Y-\frac{\partial p}{\partial y}=-\frac{\partial}{\partial y}(\Omega+p), Z-\frac{\partial p}{\partial z}=-\frac{\partial}{\partial z}(\Omega+p)$$

So p is adequately supplanted by p +ω If X, Y, Z are known or are equations (2) – (4) give an arrangement of four coupled nonlinear partial differential equations for the four obscure function, u, v, w, and p.

3. MASS-BALANCE EQUATIONS: FIRST METHOD OF GETTING PDE MODELS

(i) Equation of Continuity in Fluid Dynamics:

In the event that V_n is the typical segment of the velocity of the fluid anytime of the outside of our calculated volume component (Fig. 1), the mass of the fluid flowing out in time Δt over the surface

$$=\Delta t\iint_S \rho V_n dS=\Delta t\iint_S \rho\vec{V}.d\vec{S} \quad (6)$$

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$$= \Delta t \iiint_T div(\rho \vec{V}) dx\, dy\, dz, \tag{7}$$

On utilizing Gauss' Divergence Theorem the difference in mass of liquid in the volume component in the time Δt is given by

$$-\Delta t \frac{\partial}{\partial t} \iiint_T \rho\, dx\, dy\, dz = -\Delta t \iiint_T \frac{\partial \rho^0}{\partial t} dx\, dy\, dz \tag{8}$$

Using (6) and (7), the principle of mass-balance gives

$$\iiint_T \left[\frac{\partial \rho}{\partial t} - div(\rho \vec{V}) \right] dx\, dy\, dz = 0 \tag{9}$$

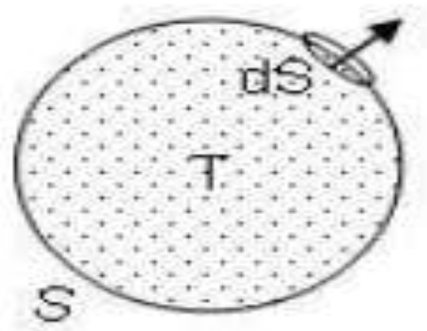

Figure 1: fluid flowing out in time Δt

Since (8) is to be true for all arbitrary volume elements, we get

$$\frac{\partial \rho}{\partial t} + div\ (\rho \vec{V}) = 0 \tag{10}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{11}$$

If the fluid is incompressible, P is constant and (9), (10) give

$$div(\vec{V})=0 \quad \text{or} \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (12)$$

Further if the flow is irrotational for example in the event that there exists a scalar velocity potential function Φ with the end goal that

$$\vec{V}=-grad\,\Phi \qquad or \qquad u=-\frac{\partial \Phi}{\partial x},$$

$$v=-\frac{\partial \Phi}{\partial y}, \qquad w=-\frac{-\partial \Phi}{\partial x}, \qquad (13)$$

Then equation (12) and (13) give

$$\Delta^2\Phi=0 \qquad or \qquad \frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \qquad (14)$$

Hence the velocity potential for irrotational stream fulfills the Laplace equation and is a harmonic function.

4.MOMENTUM-BALANCE EQUATIONS: THE-SECOND METHOD OF OBTAINING PARTIAL DIFFERENTIAL EQUATION MODELS

- **Partial Differential Equation Model for a Vibrating String:**

Leave T alone the tension of the elastic string held firmly between the focuses A and B relating to x = 0 and x = L. Leave the string alone somewhat upset. Let u(x, t) be the dislodging at time t of a component of unique length Δx and mass ρΔx. The power on this component toward the dislodging (Fig. 2)

$$=(T\,Sin\,\psi)_{x+\Delta}-(T\,\sin\psi)_x$$

$$=f(x+\Delta x)-f(x); \qquad f(x)=T\,\sin\psi$$

$$\equiv \Delta x f'(x)=\Delta x\frac{\partial}{\partial x}(T\,\sin\,\psi)$$

$$\cong \Delta x\frac{\partial}{\partial x}(T\tan\psi)=\Delta x\frac{\partial}{\partial x}\left(T\frac{\partial u}{\partial x}\right)=\Delta x T\frac{\partial^2 u}{\partial x^2}, \qquad (15)$$

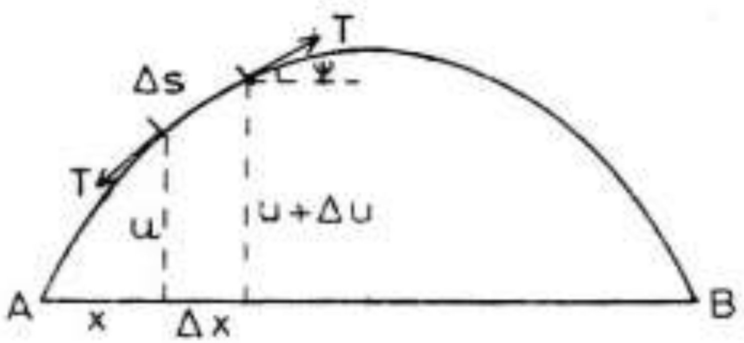

Figure 2: the direction of the displacement

So that the equation of motion for this element is

$$\rho \Delta x \frac{\partial^2 u}{\partial t^2} = T \frac{\partial^2 u}{\partial x^2} \Delta x$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}; \qquad c^2 = \frac{T}{\rho} \tag{16}$$

This is the wave equation in one dimension.**5.VARIATIONAL PRINCIPLES: THIRD METHOD OF OBTAINING PARTIAL DIFFERENTIAL EQUATION MODELS**

- **Euler-Lagrange Equation:**

Let,

$$I = \iint_S F(x, y, u, u_x, u_y)\, dx\, dy \tag{17}$$

Where F ( ) is a known function, at that point the estimation of *I* relies upon u(x, y) and our object is to pick u(x, y) with the goal that the basic *I* has a most extreme or least worth. Such a function is given by Euler-Lagrange equation of calculus of varieties viz.

$$\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0 \tag{18}$$

Since F is a known function of x, y, u, $u_x u_y$, in this way$\partial F/ \partial u$ $\partial F/\partial u_x$, $\partial F/\partial u_y$, are likewise known functions of x, y, u, u_x, w_y. As such the left hand side of (18) is a known function of x, y, u, u_x, u_y, u_Xx, u_{xy}, u_{yy} so that (18) gives a partial differential condition of second order for deciding u(x, y).

6. CONCLUSION

Mathematical modeling through differential equations are helpful for getting huge findings from models and for acquiring profound information and understanding into different entangled issues of engineering. Partial differential condition (PDE) models emerge when the variables of intrigue are functions of more than one independent variable and all the dependent and independent variables are nonstop. The mass of a volume component increased by its speeding up Vector is equivalent to the vector aggregate of all the outside body forces following up on the volume component and the internal forces because of the activity of the remainder of the substance on the volume component viable, we get straightforwardly a partial differential condition.

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